iFM & ABZ 2012 Tutorial
Safety, Dependability and Performance Analysis of Extended AADL Models
Part 6: Performability Evaluation

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Contents

Outline

Introduction

Error models

AADL error models are finite automata enriched with probabilistic failures and repairs.

Two kinds of error models can be distinguished:

- Discrete-time
  - Failures and repairs are modeled by discrete probabilities
  - Instantaneous probabilistic decision to fail (or repair)

  $\Rightarrow$ discrete-time Markov chains (DTMCs)
- Continuous-time
  - Failures and repairs are modeled by continuous probabilities
  - Mostly exponential distributions
  - Failures and repairs occur after a random duration

  $\Rightarrow$ continuous-time Markov chains (CTMCs)

As error models are interweaved with non-probabilistic nominal models, in fact decision processes result. We consider deterministic decision processes.

Outline

Let's start easy

Discrete-time Markov chain

A DTMC $\mathcal{D}$ is a tuple $(S, \mathbf{P}, \iota_{\text{init}})$ with:

- S is a countable nonempty set of states
- $\mathbf{P} : S \times S \to [0,1]$, transition probability function s.t. $\sum_{s'} \mathbf{P}(s,s') = 1$
- $\iota_{\text{init}} : S \to [0,1]$, the initial distribution with $\sum_{s \in S} \iota_{\text{init}}(s) = 1$

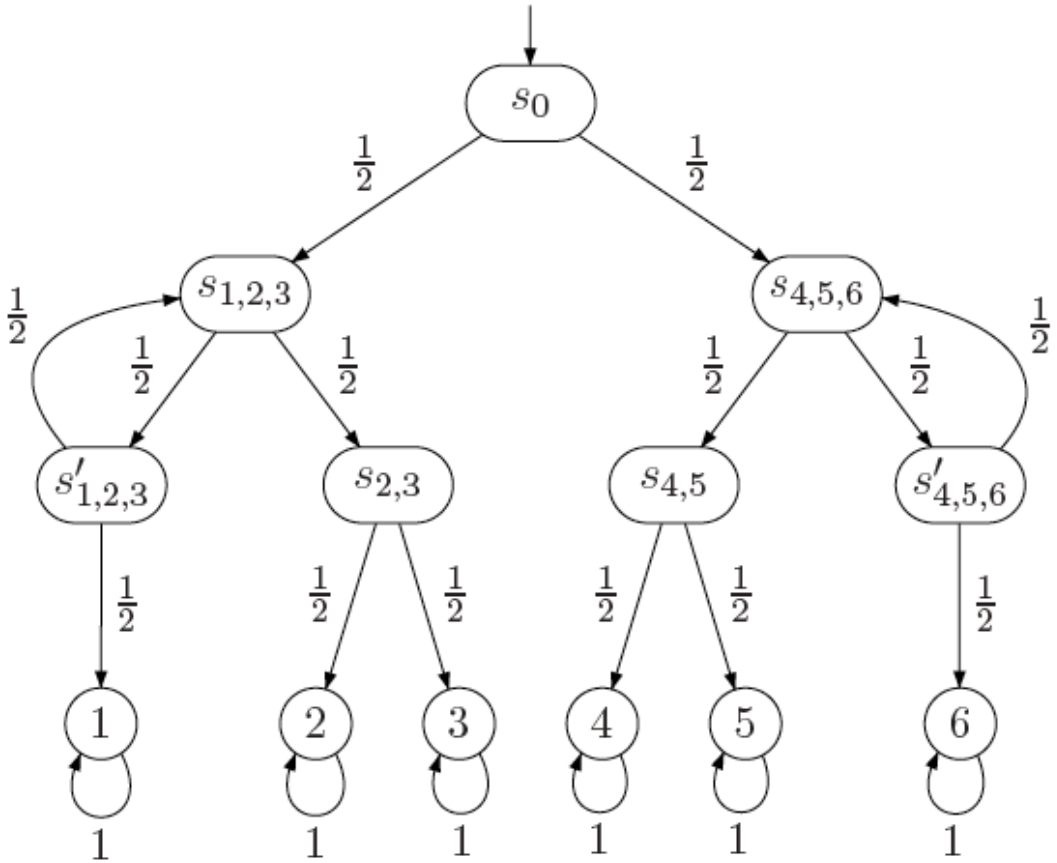
Initial states

- the set $\{ s \in S \mid \iota_{\text{init}}(s) > 0 \}$ are the possible initial states.

Paths

Paths through a DTMC are infinite sequence of states.

Simulating a die by a fair coin [Knuth & Yao]



Heads = "go left"; tails = "go right". Does this DTMC adequately model a fair six-sided die?

Some events of interest

(Simple) reachability

Eventually reach a state in $G \subseteq S$. Formally:

$$\Diamond G \;=\; \{\, \pi \in \mathit{Paths}(\mathcal{D}) \mid \exists i \in \mathbb{N}.\, \pi[i] \in G \,\}$$

Invariance, i.e., always stay in state in G:

$$\Box G \;=\; \{\, \pi \in \mathit{Paths}(\mathcal{D}) \mid \forall i \in \mathbb{N}.\, \pi[i] \in G \,\} \;=\; \overline{\Diamond \overline{G}}.$$

Constrained reachability

Or "reach-avoid" properties where states in $F \subseteq S$ are forbidden:

$$\overline{F}\,\mathsf{U}\,G \;=\; \{\, \pi \in \mathit{Paths}(\mathcal{D}) \mid \exists i \in \mathbb{N}.\, \pi[i] \in G \wedge \forall j < i.\, \pi[j] \notin F \,\}$$

In a similar way, $\Box\Diamond G$ and $\Diamond\Box G$ are defined.

Reachability probabilities in finite DTMCs

Problem statement

Let $\mathcal{D}$ be a DTMC with finite state space S, $s \in S$ and $G \subseteq S$.

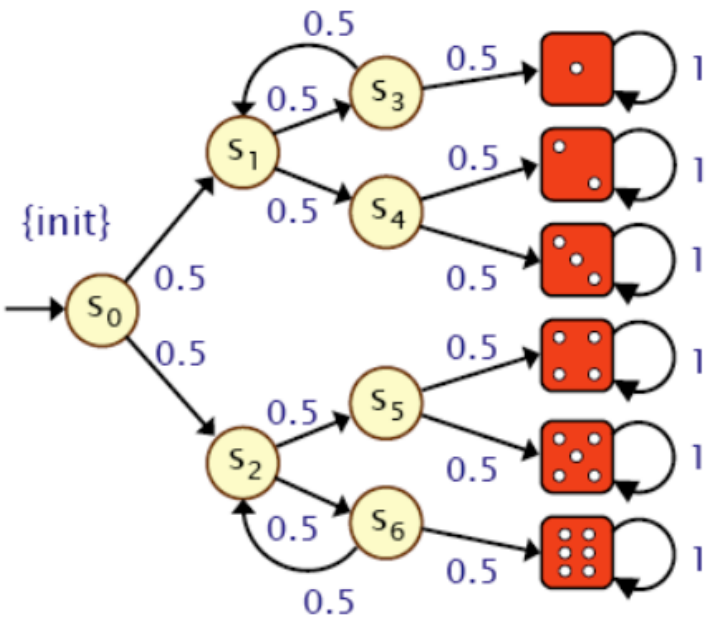
Aim: determine $Pr(s \models \Diamond G) = Pr_s\{\, \pi \in Paths(s) \mid \pi \in \Diamond G \,\}$.

Characterisation of reachability probabilities

- Let variable $x_s = Pr(s \models \Diamond G)$ for any state s
  - if G is not reachable from s, then $x_s = 0$
  - if $s \in G$ then $x_s = 1$
- For any state $s \in Pre^*(G) \setminus G$:

$$x_s = \underbrace{\sum_{t \in S \setminus G} \mathbf{P}(s,t) \cdot x_t}_{\text{reach } G \text{ via } t \in S \setminus G} + \underbrace{\sum_{u \in G} \mathbf{P}(s,u)}_{\text{reach } G \text{ in one step}}$$

Reachability probabilities: Knuth's die



- Consider the event $\Diamond 4$
- Using the previous characterisation we obtain:

$$x_1 = x_2 = x_3 = x_5 = x_6 = 0 \text{ and } x_4 = 1$$

$$x_{s_1} = x_{s_3} = x_{s_4} = 0$$

$$x_{s_0} = \frac{1}{2}x_{s_1} + \frac{1}{2}x_{s_2}$$

$$x_{s_2} = \frac{1}{2}x_{s_5} + \frac{1}{2}x_{s_6}$$

$$x_{s_5} = \frac{1}{2}x_5 + \frac{1}{2}x_4$$

$$x_{s_6} = \frac{1}{2}x_{s_2} + \frac{1}{2}x_6$$

- Gaussian elimination yields:

$$x_{s_5} = \frac{1}{2},\ x_{s_2} = \frac{1}{3},\ x_{s_6} = \frac{1}{6},\ \text{and } \boxed{x_{s_0} = \frac{1}{6}}$$

Linear equation system

Reachability probabilities as linear equation system

- Let $S_? = Pre^*(G) \setminus G$, the states that can reach G by > 0 steps
- $\mathbf{A} = \big(\mathbf{P}(s,t)\big)_{s,t \in S_?}$, the transition probabilities in $S_?$
- $\mathbf{b} = \big(b_s\big)_{s \in S_?}$, the probs to reach G in 1 step, i.e., $b_s = \sum_{u \in G} \mathbf{P}(s,u)$

Then: $\mathbf{x} = (x_s)_{s \in S_?}$ with $x_s = Pr(s \models \Diamond G)$ is the unique solution of:

$$\mathbf{x} \;=\; \mathbf{A}{\cdot}\mathbf{x} \;+\; \mathbf{b} \quad \text{or} \quad (\mathbf{I} - \mathbf{A}){\cdot}\mathbf{x} \;=\; \mathbf{b}$$

where $\mathbf{I}$ is the identity matrix of cardinality $|S_?| \times |S_?|$.

Repeated reachability and persistence

Long-run theorem

Almost surely any finite DTMC eventually reaches a BSCC and visits all its states infinitely often.

Repeated reachability = Reachability

For finite DTMC with state space S, $G \subseteq S$, and $s \in S$:

$$Pr(s \models \Box\Diamond G) \;=\; Pr(s \models \Diamond U)$$

where U is the union of all BSCCs T with $T \cap G \neq \emptyset$.

Persistency = Reachability

For finite DTMC with state space S, $G \subseteq S$, and $s \in S$:

$$Pr(s \models \Diamond\Box G) \;=\; Pr(s \models \Diamond U)$$

where U is the union of all BSCCs T with $T \subseteq G$.

Probabilistic bisimulation: intuition

Intuition

- Strong bisimulation is used to compare labeled transition systems.
- Strongly bisimilar states exhibit the same step-wise behaviour.
- Our aim: adapt bisimulation to discrete-time Markov chains.
- This yields a probabilistic variant of strong bisimulation.

- When do two DTMC states exhibit the same step-wise behaviour?
- Key: if their transition probability for each equivalence class coincides.

Probabilistic bisimulation

Probabilistic bisimulation [Larsen & Skou, 1989]

Let $\mathcal{D} = (S, \mathbf{P}, \iota_{\text{init}})$ be a DTMC and $R \subseteq S \times S$ an equivalence. R is a *probabilistic bisimulation* on S if for any $(s, t) \in R$:

$$\mathbf{P}(s, C) = \mathbf{P}(t, C) \text{ for all equivalence classes } C \in S/R.$$

where $\mathbf{P}(s, C) = \sum_{s' \in C} \mathbf{P}(s, s')$.

For states in R, the probability of moving by a single transition to some equivalence class is equal.

Probabilistic bisimilarity

Let $\mathcal{D}$ be a DTMC and s, t states in $\mathcal{D}$. Then: s is *probabilistically bisimilar* to t, denoted $s \sim_p t$, if there exists a probabilistic bisimulation R with $(s, t) \in R$.

Probabilistic bisimulation

Probabilistic bisimulation

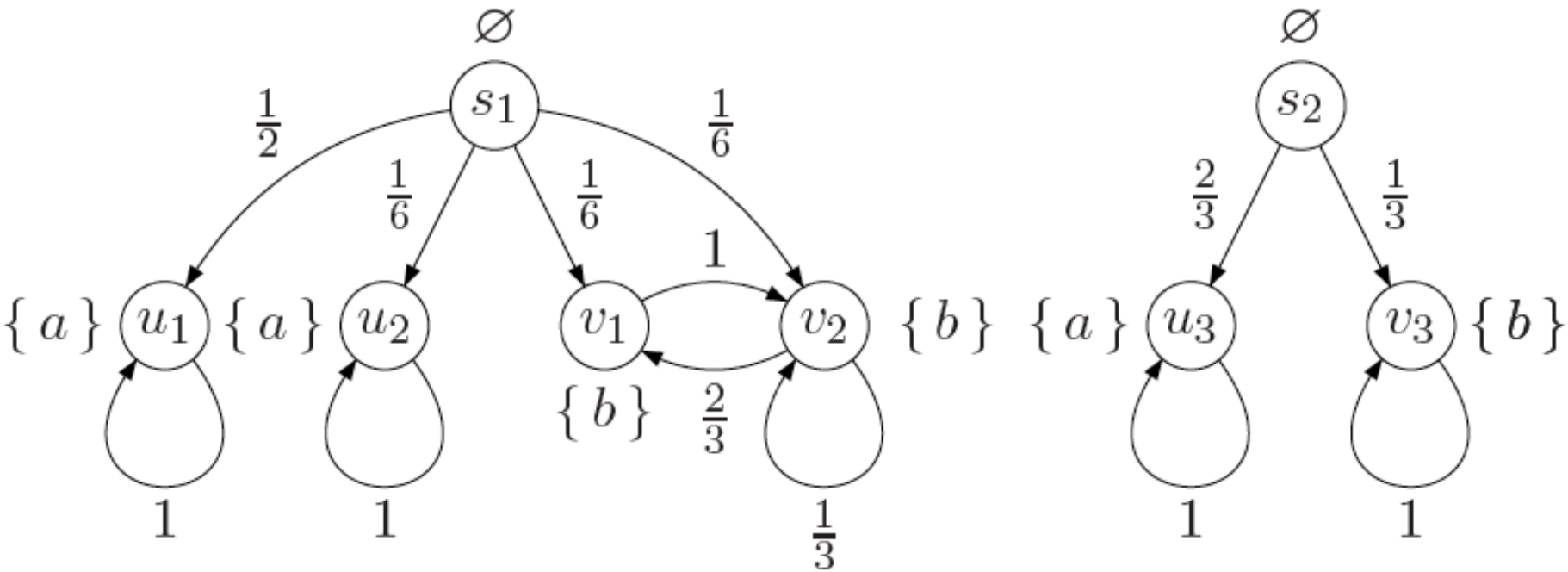
Let $\mathcal{D} = (S, \mathbf{P}, \iota_{\text{init}})$ be a DTMC and $R \subseteq S \times S$ an equivalence. Then: R is a *probabilistic bisimulation* on S if for any $(s, t) \in R$:

$$\mathbf{P}(s, C) = \mathbf{P}(t, C) \text{ for all equivalence classes } C \in S/R.$$

Remarks

As opposed to bisimulation on states in transition systems, any probabilistic bisimulation is an equivalence.

Example


∅
s_1
$\frac{1}{2}$
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1
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s_2
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1
1

Quotient under $\sim_p$

Quotient DTM under $\sim_p$

For $\mathcal{D} = (S, \mathbf{P}, \iota_{\text{init}})$ and probabilistic bisimulation $\sim_p \subseteq S \times S$ let

$$\mathcal{D}/\sim_p \;=\; (S', \mathbf{P}', \iota'_{\text{init}}), \qquad \text{the } \textit{quotient} \text{ of } \mathcal{D} \text{ under } \sim_p$$

where

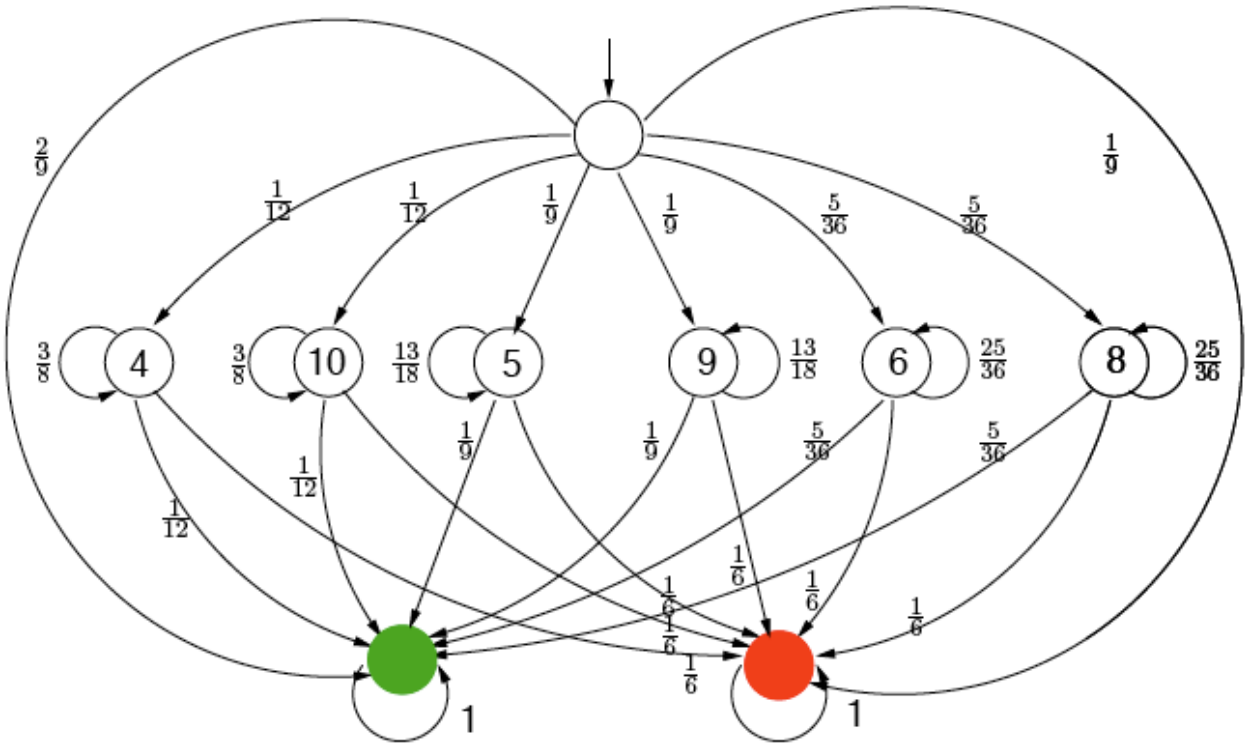
- $S' = S/\sim_p = \{\, [s]_{\sim_p} \mid s \in S \,\}$ with $[s]_{\sim_p} = \{\, s' \in S \mid s \sim_p s' \,\}$
- $\mathbf{P}'([s]_{\sim_p}, [s']_{\sim_p}) = \mathbf{P}(s, [s']_{\sim_p})$
- $\iota'_{\text{init}}([s]_{\sim_p}) = \sum_{s' \in [s]_{\sim_p}} \iota_{\text{init}}(s)$

Remarks

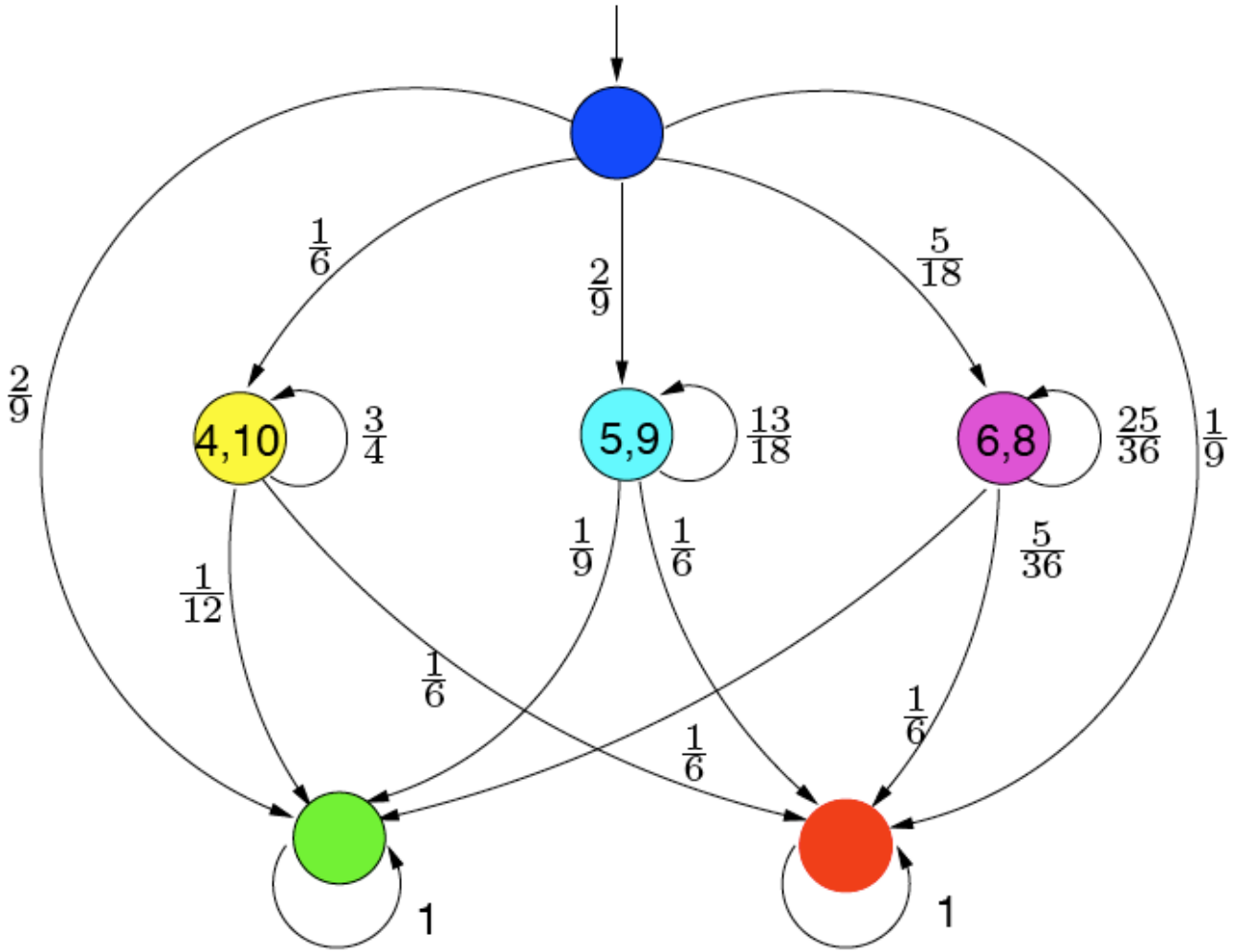
The transition probability from $[s]_{\sim_p}$ to $[t]_{\sim_p}$ equals $\mathbf{P}(s, [t]_{\sim_p})$. This is well-defined as $\mathbf{P}(s, C) = \mathbf{P}(s', C)$ for all $s \sim_p s'$ and all $\sim_p$ equivalence classes C.

Craps

- Come-out roll:
  - 7 or 11: win
  - 2, 3, or 12: lose
  - else: roll again
- Next roll(s):
  - 7: lose
  - point: win
  - else: roll again



Bisimulation quotient DTMC of Craps



Preservation

Bisimulation preserves reachability probabilities

Let $\mathcal{D}$ be a DTMC and s, t states in $\mathcal{D}$. Then:

$$s \sim_p t \quad \text{implies} \quad Pr(s \models \Diamond G) = Pr(t \models \Diamond G)$$

for every $\sim_p$-closed set of states $G \subseteq S$.

Remarks

$s \sim_p t$ implies that (repeated) reachability probabilities, and persistence probabilities for s and t coincide.

In fact, $\sim_p$ coincides with probabilistic CTL equivalence.

The coarsest bisimulation quotient can be computed in $\mathcal{O}(m \cdot \log n)$

Outline

Random timing

Negative exponential distribution

Density of exponential distribution

The density of an *exponentially distributed* r.v. Y with *rate* $\lambda \in \mathbb{R}_{>0}$ is:

$$f_Y(x) = \lambda \cdot e^{-\lambda \cdot x} \quad \text{for } x > 0 \quad \text{and } f_Y(x) = 0 \text{ otherwise}$$

The cumulative distribution of r.v. Y with rate $\lambda \in \mathbb{R}_{>0}$ is:

$$F_Y(d) = \int_0^d \lambda \cdot e^{-\lambda \cdot x}\, dx = [-e^{-\lambda \cdot x}]_0^d = 1 - e^{-\lambda \cdot d}.$$

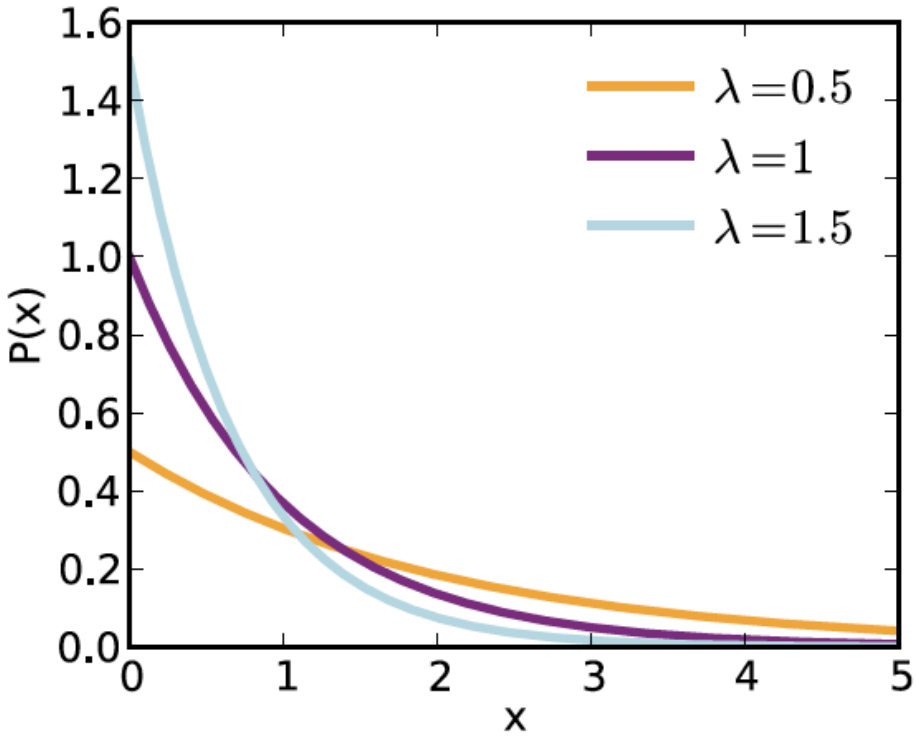
The rate $\lambda \in \mathbb{R}_{>0}$ uniquely determines an exponential distribution.

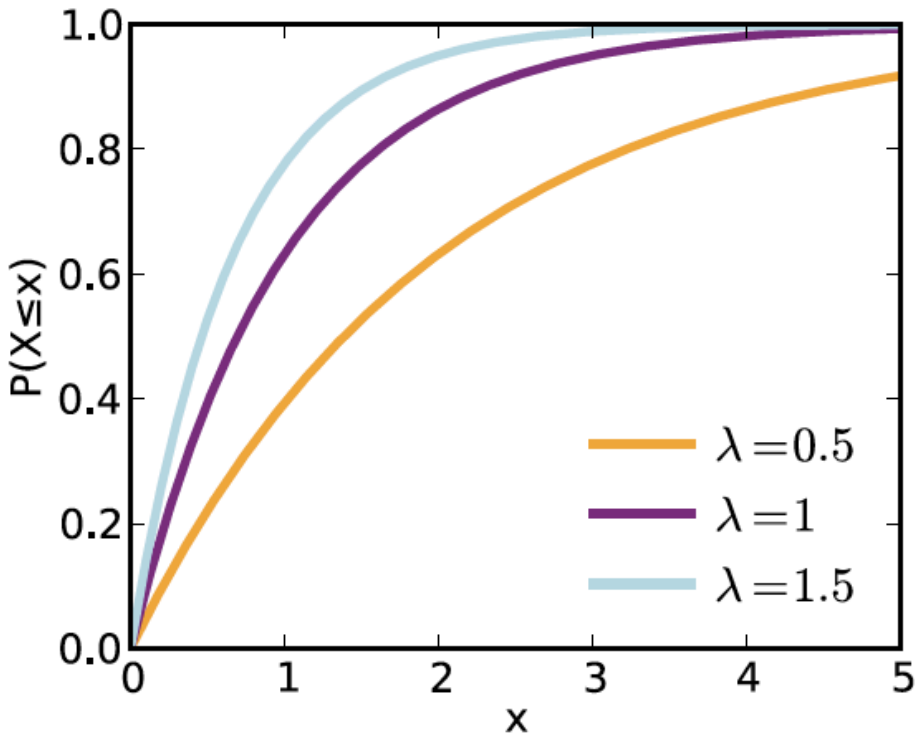
Variance and expectation

Let r.v. Y be exponentially distributed with rate $\lambda \in \mathbb{R}_{>0}$. Then:

Expectation $E[Y] = \frac{1}{\lambda}$ and variance $X[Y] = \frac{1}{\lambda^2}$

Exponential pdf and cdf





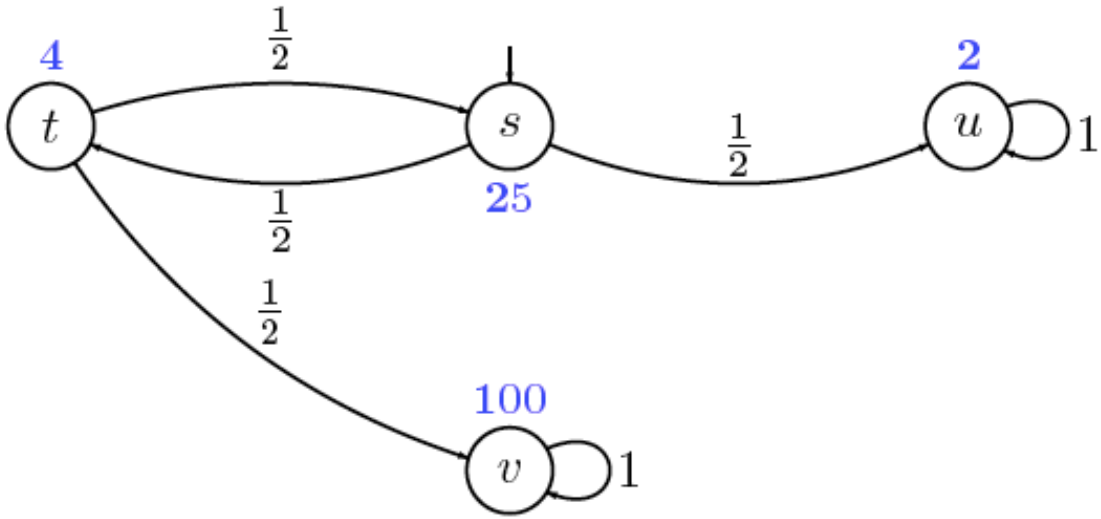
The higher λ, the faster the cdf approaches 1.

Why exponential distributions?

- Are *adequate* for many real-life phenomena
  - the time until a radioactive particle decays
  - the time between successive car accidents
  - inter-arrival times of jobs, telephone calls in a fixed interval
- Are the continuous counterpart of the *geometric* distribution
- Heavily used in physics, performance, and reliability analysis
- Can *approximate* general distributions arbitrarily closely
- Yield a *maximal entropy* if only the mean is known

Continuous-time Markov chains

A CTMC is a DTMC with an *exit rate* function $r : S \to \mathbb{R}_{>0}$ where $r(s)$ is the rate of an exponential distribution.

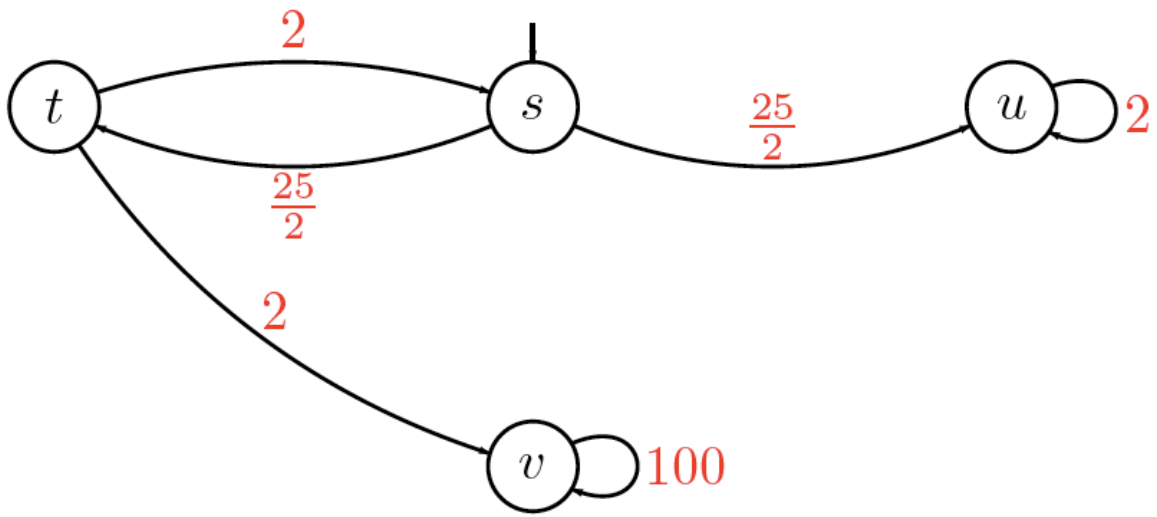


$r(s) = 25$, $r(t) = 4$, $r(u) = 2$ and $r(v) = 100$

Example: a classical perspective

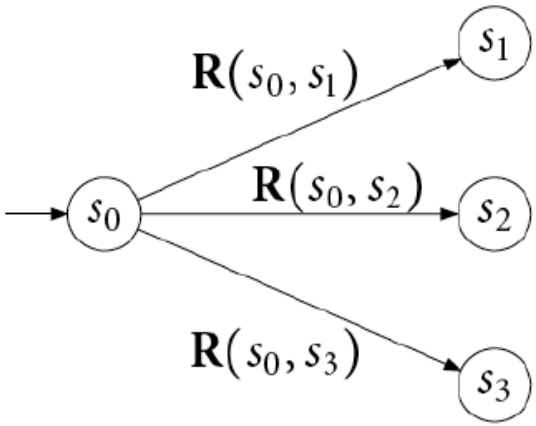
A CTMC is a DTMC with an *exit rate* function $r : S \to \mathbb{R}_{>0}$ where $r(s)$ is the rate of an exponential distribution.

A CTMC is a DTMC where transition probability function **P** is replaced by a *transition rate* function **R**. We have $\mathbf{R}(s, s') = \mathbf{P}(s, s') \cdot r(s)$.



$r(s) = 25$, $r(t) = 4$, $r(u) = 2$ and $r(v) = 100$

CTMC semantics by example



CTMC semantics

- Transition $s \to s'$:= r.v. $X_{s,s'}$ with rate $\mathbf{R}(s, s')$
- Probability to go from state s_0 to, say, state s_2 is:

$$Pr\{X_{s_0,s_2} \leq X_{s_0,s_1} \cap X_{s_0,s_2} \leq X_{s_0,s_3}\}$$

$$=$$

$$\frac{\mathbf{R}(s_0, s_2)}{\mathbf{R}(s_0, s_1) + \mathbf{R}(s_0, s_2) + \mathbf{R}(s_0, s_3)} = \frac{\mathbf{R}(s_0, s_2)}{r(s_0)}$$

- Probability of staying at most t time in s_0 is:

$$Pr\{\min(X_{s_0,s_1}, X_{s_0,s_2}, X_{s_0,s_3}) \leq t\}$$

$$=$$

$$1 - e^{-(\mathbf{R}(s_0,s_1)+\mathbf{R}(s_0,s_2)+\mathbf{R}(s_0,s_3))\cdot t} = 1 - e^{-r(s_0)\cdot t}$$

CTMC semantics

State-to-state timed transition probability

The probability to *move* from non-absorbing s to s' in $[0, t]$ is:

$$\frac{\mathbf{R}(s, s')}{r(s)} \cdot \left(1 - e^{-r(s)\cdot t}\right).$$

Residence time distribution

The probability to *take some* outgoing transition from s in $[0, t]$ is:

$$\int_0^t r(s)\cdot e^{-r(s)\cdot x}\, dx \;=\; 1 - e^{-r(s)\cdot t}$$

CTMCs are omnipresent!

- Markovian queueing networks (Kleinrock 1975)
- Stochastic Petri nets (Molloy 1977)
- Stochastic activity networks (Meyer & Sanders 1985)
- Stochastic process algebra (Herzog *et al.*, Hillston 1993)
- Probabilistic input/output automata (Smolka *et al.* 1994)
- Calculi for biological systems (Priami *et al.*, Cardelli 2002)

CTMCs are one of the most prominent models in performance analysis

Paths in a CTMC

Timed paths

Paths in CTMC $\mathcal{C}$ are maximal (i.e., infinite) paths of alternating states and time instants:

$$\pi \;=\; s_0 \xrightarrow{t_0} s_1 \xrightarrow{t_1} s_2 \cdots$$

such that $s_i \in S$ and $t_i \in \mathbb{R}_{>0}$.

Time instant t_i is the amount of time spent in state s_i.

Notations

- Let $\pi[i] := s_i$ denote the $(i{+}1)$-st state along the timed path π.
- Let $\pi@t$ be the state occupied in π at time $t \in \mathbb{R}_{\geq 0}$, i.e. $\pi@t := \pi[i]$ where i is the smallest index such that $\sum_{j=0}^{i} t_j > t$.

Zeno theorem

Zeno path

Path $s_0 \xrightarrow{t_0} s_1 \xrightarrow{t_1} s_2 \xrightarrow{t_2} s_3 \ldots\ldots$ is called Zeno [a] if $\sum_i t_i$ converges.

[a] Zeno of Elea (490-430 BC), philosopher, famed for his paradoxes.

Example

$$s_0 \xrightarrow{1} s_1 \xrightarrow{\frac{1}{2}} s_2 \xrightarrow{\frac{1}{4}} s_3 \ldots s_i \xrightarrow{\frac{1}{2^i}} s_{i+1} \ldots$$

In timed automata, such executions are typically excluded from the analysis.

Zeno theorem

For all states s in any CTMC, $Pr\{\, \pi \in Paths(s) \mid \pi \text{ is Zeno} \,\} = 0$.

Timed reachability events

Let CTMC $\mathcal{C}$ with (possibly infinite) state space S.

(Simple) timed reachability

Eventually reach a state in $G \subseteq S$ in the interval I. Formally:

$$\Diamond^I G = \{ \pi \in Paths(\mathcal{C}) \mid \exists t \in I . \pi@t \in G \}$$

Invariance, i.e., always stay in state in G in the interval I:

$$\Box^I G = \{ \pi \in Paths(\mathcal{C}) \mid \forall t \in I . \pi@t \in G \} = \overline{\Diamond^I \overline{G}}.$$

Constrained timed reachability

Or "reach-avoid" properties where states in $F \subseteq S$ are forbidden:

$$\overline{F}\, \mathsf{U}^I\, G = \{ \pi \in Paths(\mathcal{C}) \mid \exists t \in I . \pi@t \in G \wedge \forall d < t . \pi@d \notin F \}$$

Measurability

Measurability theorem

Events $\Diamond^I G$, $\Box^I G$, and $\overline{F}\, \mathsf{U}^I G$ are measurable on any CTMC.

Timed reachability probabilities in finite CTMCs

Problem statement

Let $\mathcal{C}$ be a CTMC with finite state space S, $s \in S$, $t \in \mathbb{R}_{\geq 0}$ and $G \subseteq S$.

Aim: $Pr(s \models \Diamond^{\leq t} G) = Pr_s\{ \pi \in Paths(s) \mid \pi \models \Diamond^{\leq t} G \}$

where Pr_s is the probability measure in CTMC $\mathcal{C}$ with single initial state s.

Characterisation of timed reachability probabilities

- Let function $x_s(t) = Pr(s \models \Diamond^{\leq t} G)$ for any state s
  - if G is not reachable from s, then $x_s(t) = 0$ for all t
  - if $s \in G$ then $x_s(t) = 1$ for all t
- For any state $s \in Pre^*(G) \setminus G$:

$$x_s(t) = \int_0^t \sum_{s' \in S} \underbrace{\mathbf{R}(s,s') \cdot e^{-r(s) \cdot x}}_{\text{probability to move to state } s' \text{ at time } x} \cdot \underbrace{x_{s'}(t-x)}_{\text{prob. to fulfill } \Diamond^{\leq t-x} G \text{ from } s'} \, dx$$

Reachability

Reachability probabilities in finite DTMCs and CTMCs

Solve a system of linear equations (using some efficient techniques).

Timed reachability probabilities in finite CTMCs

Solve a system of Volterra integral equations. This is in general non-trivial, inefficient, and has several pitfalls such as numerical stability.

Solution

Reduce the problem of computing $Pr(s \models \Diamond^{\leq t} G)$ to an alternative problem for which well-known efficient techniques exist: computing transient probabilities.

Timed reachability probabilities = transient probabilities

Aim

Compute $Pr(s \models \Diamond^{\leq t} G)$ in CTMC $\mathcal{C}$. Observe that once a path π reaches G within t time, then the remaining behaviour along π is not important. This suggests to make all states in G absorbing.

Let CTMC $\mathcal{C} = (S, \mathbf{P}, r, \iota_{\text{init}})$ and $G \subseteq S$. The CTMC $\mathcal{C}[G] = (S, \mathbf{P}_G, r, \iota_{\text{init}})$ with $\mathbf{P}_G(s,t) = \mathbf{P}(s,t)$ if $s \notin G$ and $\mathbf{P}_G(s,s) = 1$ if $s \in G$.

All outgoing transitions of $s \in G$ are replaced by a single self-loop at s.

Lemma

$$\underbrace{Pr(s \models \Diamond^{\leq t} G)}_{\text{timed reachability in } \mathcal{C}} = \underbrace{Pr(s \models \Diamond^{=t} G)}_{\text{timed reachability in } \mathcal{C}[G]} = \underbrace{\underline{p}(t) \text{ with } \underline{p}(0) = \mathbf{1}_s}_{\text{transient prob. in } \mathcal{C}[G]}.$$

Transient distribution theorem

Theorem: transient distribution as ordinary differential equation

The transient probability vector $\underline{p}(t) = (p_{s_1}(t), \ldots, p_{s_k}(t))$ satisfies:

$$\underline{p}'(t) \;=\; \underline{p}(t) \cdot (\mathbf{R} - \mathbf{r}) \quad \text{given} \quad \underline{p}(0)$$

where $\mathbf{r}$ is the diagonal matrix of vector $\underline{r}$.

Solution technique:

Transform the CTMC (again), and then truncate a Taylor-MacLaurin expansion. This yields a polynomial-time approximation algorithm.

Bisimulation on CTMCs

Probabilistic bisimulation [Buchholz, 1994]

Let $\mathcal{C} = (S, \mathbf{P}, r, \iota_{\text{init}})$ be a CTMC and $R \subseteq S \times S$ an equivalence. Then: R is a *probabilistic bisimulation* on S if for any $(s, t) \in R$:

1. $r(s) = r(t)$, and
2. $\mathbf{P}(s, C) = \mathbf{P}(t, C)$ for all equivalence classes $C \in S/R$

The last two conditions amount to $\mathbf{R}(s, C) = \mathbf{R}(t, C)$ for all equivalence classes $C \in S/R$.

Probabilistic bisimilarity

Let $\mathcal{C}$ be a CTMC and s, t states in $\mathcal{C}$. Then: s is *probabilistically bisimilar* to t, denoted $s \sim_m t$, if there exists a probabilistic bisimulation R with $(s, t) \in R$.

Preservation

Bisimulation preserves timed reachability probabilities

Let $\mathcal{C}$ be a CTMC and s, t states in $\mathcal{C}$. Then:

$$s \sim_m t \quad \text{implies} \quad Pr(s \models \Diamond^{\leq t} G) = Pr(t \models \Diamond^{\leq t} G)$$

for every $\sim_p$-closed set of states $G \subseteq S$ and any t.

Remarks

$s \sim_p t$ implies that (repeated) reachability probabilities, and persistence probabilities for s and t coincide.

In fact, $\sim_p$ coincides with probabilistic CTL equivalence.

The coarsest bisimulation quotient can be computed in $\mathcal{O}(m \cdot \log n)$

Outline

Approach

Approach in the COMPASS toolset

1. Weave the nominal behaviour and error model (model extension)
2. The semantics yields an continuous-time decision process
3. Apply (BDD-based) bisimulation minimisation to this process
4. Mostly this yields a CTMC
5. Verify it using the techniques explained before
6. For timed reachability, cover the entire range from 0 to t

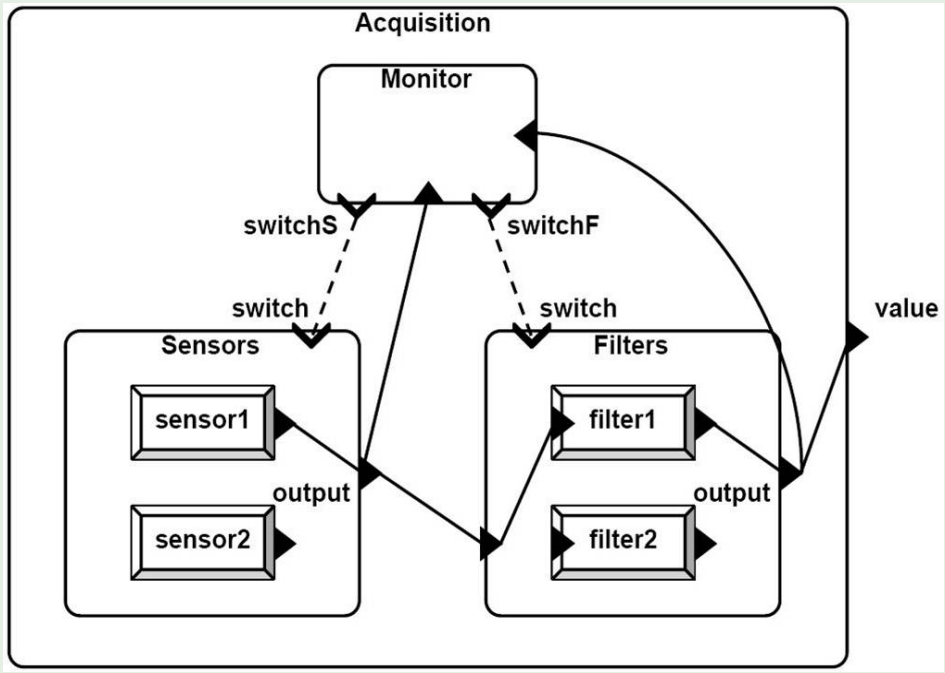
Current work is on directly analysing the stochastic decision process

Demo Example: Sensor-Filter Acquisition System

Redundant Sensor-Filter Example: Nominal Model

- models a value acquisition system
- the value is read by a sensor, filtered by a filter, and returned as output
- two redundant sensors `sensor1` and `sensor2`
- two redundant filters `filter1` and `filter2`
- a central `Monitor` detects anomalies in either the output of the sensor or the filter, and issues a system reconfiguration (`switchS` or `switchF`) whenever needed

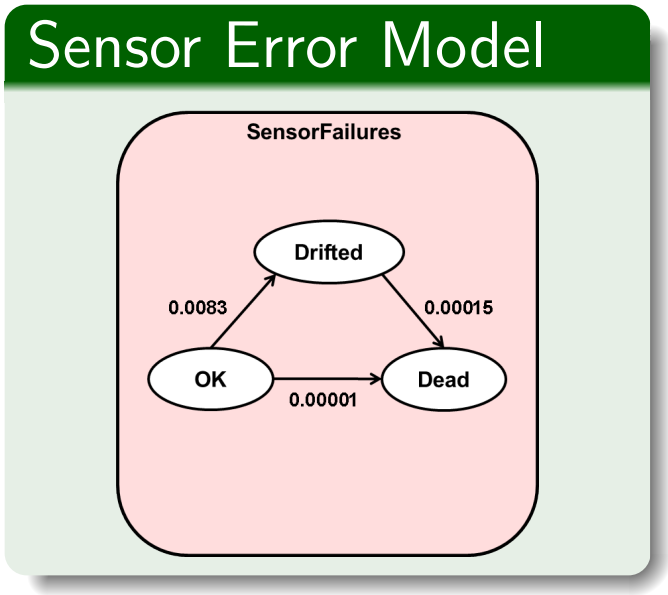
Acquisition System



Modeling Errors

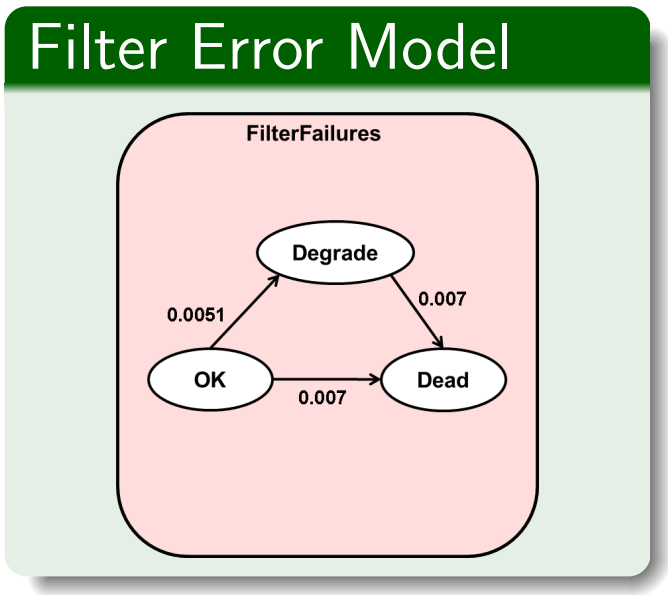
Sensor Error model:

- two faulty states: `Drifted` and `Dead`
- poisson distribution



Filter Error model:

- two faulty states: `Degrade` and `Dead`
- poisson distribution

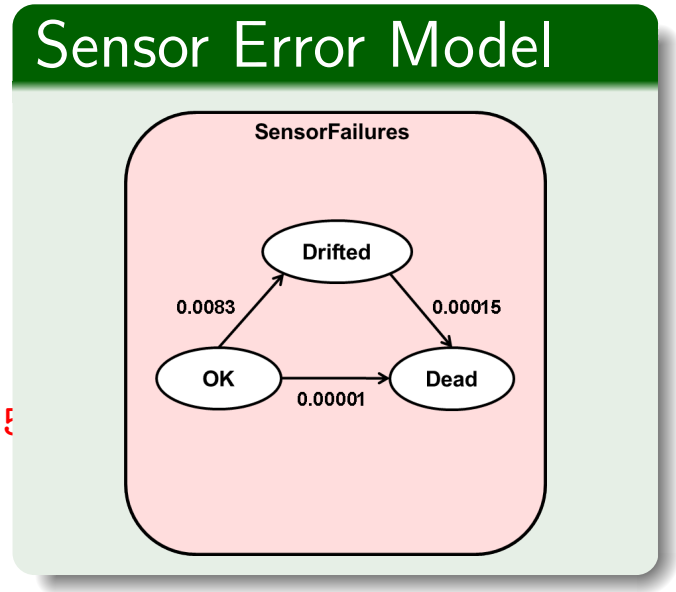


Modeling Errors

Sensor: SLIM Error Model

```
error model SensorFailures
  features
    OK: initial state;
    Drifted: error state;
    Dead: error state;
end SensorFailures;


error model implementation SensorFailures.Impl
  events
    drift: error event occurrence poisson 0.083;
    die: error event occurrence poisson 0.00001;
    dieByDrift: error event occurrence poisson 0.00015
  transitions
    OK -[ die ]-> Dead;
    OK -[ drift ]-> Drifted;
    Drifted -[ dieByDrift ]-> Dead;
end SensorFailures.Impl;
```

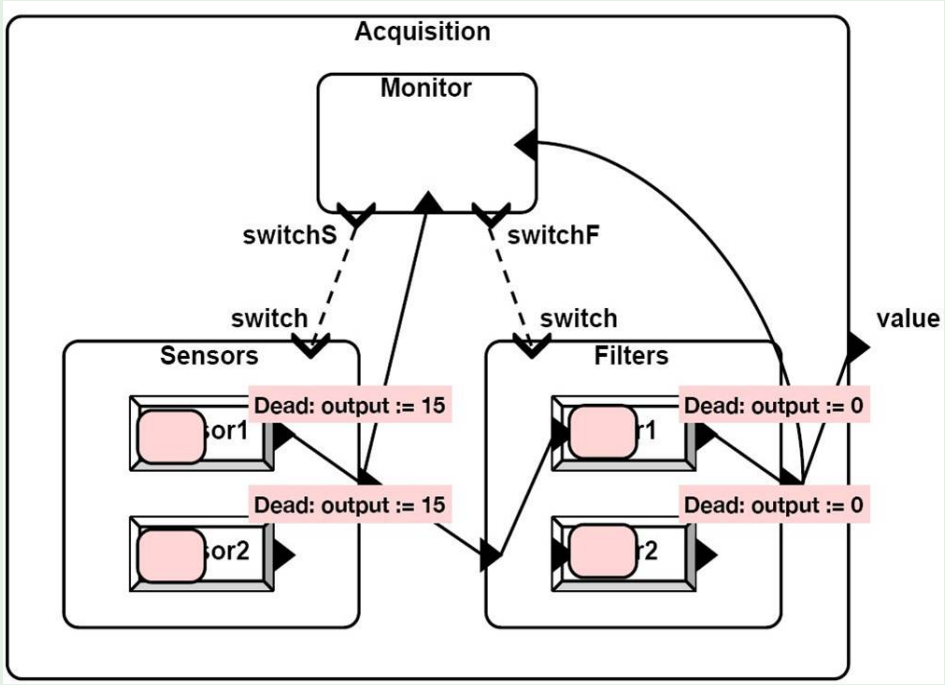


Modeling Errors

Fault Injections:

- in state Dead the output of the sensor is stuck at 15
- in state Dead the output of the filter is stuck at 0

Fault Injections



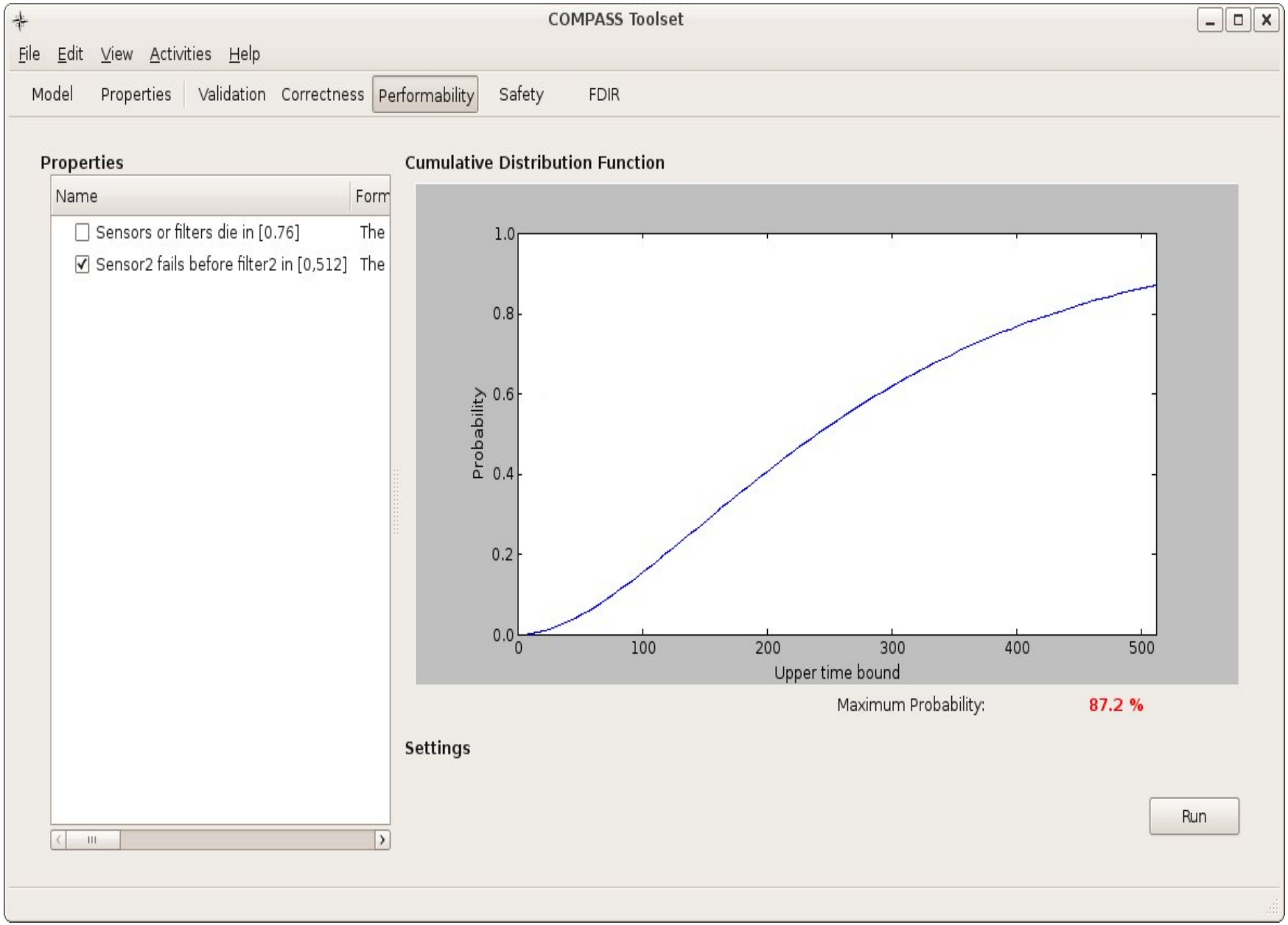
Properties of interest

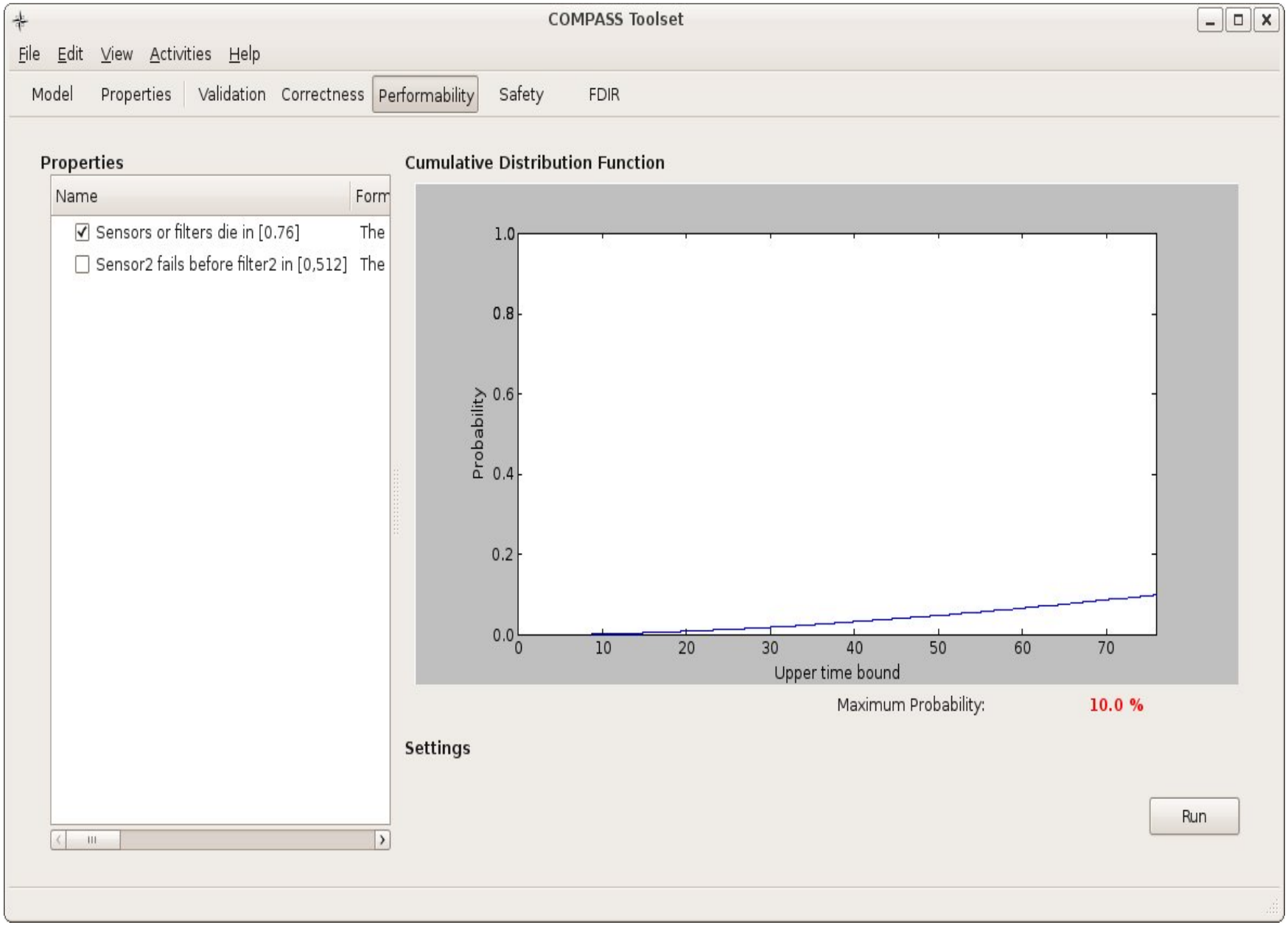
Some properties of interest

- A filter or a sensor fails
- A sensor fails
  - `sensor1` fails
  - `sensor2` fails
- Filters fail twice
- Monitor reacts to filter failures
- Sensors or filters die within 76 hours
- `sensor2` fails before `filter2` within 512 hours

Example: Sensor filter example

Recapitulate the sensor filter example with error model.


COMPASS Toolset
File Edit View Activities Help
Model Properties Validation Correctness Performability Safety FDIR
Properties
Name Form
Sensors or filters die in [0.76] The
Sensor2 fails before filter2 in [0,512] The
Cumulative Distribution Function
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0 100 200 300 400 500
Upper time bound
Maximum Probability: 87.2 %
Settings
Run


COMPASS Toolset
File Edit View Activities Help
Model Properties Validation Correctness Performability Safety FDIR
Properties
Name Form
Sensors or filters die in [0.76] The
Sensor2 fails before filter2 in [0,512] The
Cumulative Distribution Function
Probability
1.0
0.8
0.6
0.4
0.2
0.0
0 10 20 30 40 50 60 70
Upper time bound
Maximum Probability: 10.0 %
Settings
Run

Further information

- Probabilistic model checking (Baier et. al, CACM 2011) (Kwiatkowska et. al, SFM 2011) (Baier & Katoen, Principles of Model Checking)
- CTMC model checking (Baier et. al, IEEE TSE 2003)
- Probabilistic bisimulation (Larsen & Skou, Inf. Comp 1989) (Kemeny & Snell, 1960) (Buchholz, Appl. Prob. 1994)
- Bisimulation minimisation (Derisavi et. al, IPL 2005) (Valmari & Franceschinis, TACAS 2010)
- Stochastic decision processes (Guck et. al, NFM 2012)